

# COMPRESSED SENSING DYNAMIC MRI RECONSTRUCTION USING MULTI-SCALE 3D CONVOLUTIONAL SPARSE CODING WITH ELASTIC NET REGULARIZATION

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## ABSTRACT

In this paper, we introduce a fast alternating method to reconstruct highly undersampled dynamic MRI data by using multi-scale 3D convolutional sparse coding. The proposed method concurrently builds a multi-scale 3D dictionary as the MRI reconstruction proceeds by using a variant of the alternating direction method of multipliers algorithm. In addition, elastic net regularization is also applied to take the advantages of both lasso and ridge regularizations for promoting better sparse approximation to the measurement data. We demonstrate that the reconstruction quality of our method is higher than the state-of-the-art dictionary-based MRI reconstruction algorithms.

**Index Terms**— Multi-scale 3D CSC, Elastic Net Regularization, Compressed Sensing, MRI, GPU

## 1. INTRODUCTION

Dynamic MRI is widely used to analyze physiological tissue characteristics (dynamic contrast enhanced MRI) or the anatomy and functions of the heart (cardiac MRI). To increase temporal resolution of dynamic MRI by accelerating its acquisition time, compressed sensing (CS) has been actively studied and adopted into MRI [1]. Conventional CS-MRI reconstruction methods have exploited the sparsity of signal by applying universal sparsifying transforms such as Wavelet and Fourier transforms [2]. More recently, data-driven methods (i.e., dictionary learning and convolutional sparse coding) have been proposed to improve the reconstruction quality [3, 4]. On the other hand, parallel architecture (e.g., GPU) has been exploited to accelerate running time of numerical solvers [3].

Conventional dictionary learning uses a batch of small image patches for training, but this approach suffers from drawbacks such as redundant atoms and longer running times. The recent development of convolutional sparse coding (CSC) overcomes such drawbacks by convolution of filters over the entire image domain, which is further optimized by element-wise multiplication in the frequency domain in the

Alternating Direction Method of Multiplier (ADMM) framework [5, 6]. However, fixed-size atoms do not adapt well to image features in various sizes. Therefore, we introduce multi-scale 3D CSC with elastic net regularization for CS dynamic MRI, in which multiple sizes of 3D filters and elastic net regularization can give better reconstruction by capturing more sufficient information in data. Our approach shows the outstanding results in the reconstruction quality of cardiac MRI compared to single-scale 3D CSC [3] and patch-based dictionary [4]. It can also be implemented on data-parallel architecture such as GPU for efficient computation.

## 2. METHOD

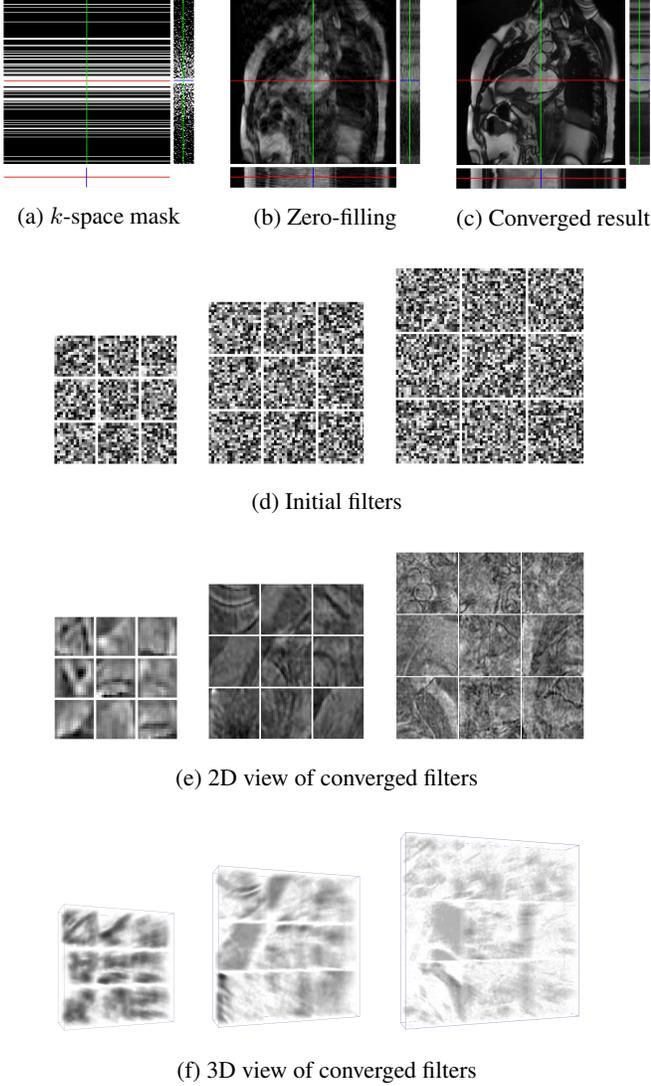
Figure 1 describes the components of the proposed CS-MRI reconstruction method. If we use an undersampling mask (Fig. 1a,  $\times 4$  undersampling) to sparsely sample the MRI  $k$ -space data and apply the inverse Fourier transform, then the resulting reconstructed images will suffer from artifacts (Fig. 1b, zero-filling reconstruction). Thus, our iterative reconstruction process with multiple different sizes of randomly initialized filters (e.g., the filter size varies between  $15 \times 15 \times 15$ ,  $30 \times 30 \times 20$ , and  $60 \times 60 \times 30$ , as shown in Fig. 1d) uses zero-filling reconstruction as an initial guess to improve the image quality. The input image and filters are iteratively updated until they converge as shown in Figs. 1c, 1e and 1f.

The proposed CS-MRI reconstruction is a process of finding  $s$  (i.e., a stack of 2D MR images for a given time duration) in the energy minimization problem as follows:

$$\begin{aligned} \min_{d, x, s} \frac{\alpha}{2} \left\| s - \sum_n \sum_k d_{n,k} * x_{n,k} \right\|_2^2 \\ + \lambda_1 \sum_n \sum_k \left\| x_{n,k} \right\|_1 + \frac{\lambda_2}{2} \sum_n \sum_k \left\| x_{n,k} \right\|_2^2 \quad (1) \\ s.t. : \left\| R\mathcal{F}_2 s - m \right\|_2^2 < \epsilon^2, \left\| d_{n,k} \right\|_2^2 \leq 1 \end{aligned}$$

where the operator  $*$  is the convolutional operator.  $N, K$  are the number of different filter sizes and the number of filters of each size respectively. The variable  $d_{n,k}$  is the filter (or atom

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**Fig. 1:** An overview of CS-MRI reconstruction using multi-scale 3D CSC

in the dictionary) of the  $k^{th}$  filter in the  $n^{th}$  dictionary size and  $x_{n,k}$  is its corresponding sparse code for  $s$ . Multiple sizes of the dictionary capture more features, which can be small or large, compared with using only a single size of dictionary, as in [3, 4]. In Eq. 1, the first term measures the difference between  $s$  and its sparse approximation  $s - \sum \sum d_{n,k} * x_{n,k}$ , weighted by  $\alpha$ . The combination of the second and third terms weighted by  $\lambda_1$  and  $\lambda_2$  parameters is called elastic net regularization, which outperforms the  $l_1$  regularization while allowing a similar sparsity of representation [7]. The rest of this equation is the collection of constraints: the first constraint keeps the consistency between the undersampled measurement  $m$  and the undersampled reconstructed image using  $k$ -space mask  $R$  with the Fourier operator  $\mathcal{F}$ ; the second constraint restricts the Frobenius norm of each atom  $d_{n,k}$  within

a unit length. In the following discussion, we simplify the notations without indices  $n, k$  and also replace the result of the Fourier transform of a given variable by using subscript  $f$  (for instance,  $d_f$  represents the simplified notation for  $\mathcal{F}d$  in 3D domain and  $s_{f_2}$  is the simplified notation for  $\mathcal{F}_2s$  in 2D spatial domain) to derive the solution of Eq. 1; then the problem 1 can be written using auxiliary variables  $y$  and  $g$  for  $x$  and  $d$  as follows:

$$\begin{aligned} \min_{d,x,g,y,s} \frac{\alpha}{2} \left\| s - \sum \sum d * x \right\|_2^2 + \lambda_1 \|y\|_1 + \frac{\lambda_2}{2} \|y\|_2^2 \\ \text{s.t. : } x - y = 0, \|R\mathcal{F}_2s - m\|_2^2 < \epsilon^2, \\ g = Proj(d), \|g\|_2^2 \leq 1 \end{aligned} \quad (2)$$

where  $g$  and  $d$  are related by a projection operator as a combination of a truncated matrix with a corresponding dictionary size, followed by a padding-zero in order to make the dimension of  $g$  the same as that of  $x$ , and the variable  $g$  should also be zero-padded to make its size similar as  $g_f$  and  $x_f$  so that we can leverage the Fourier transform to solve this problem. The constrained Eq. 2 can be unconstrained by using dual variables  $u, h$ , and further regulates the measurement consistency and the dual differences with  $\gamma, \rho, \sigma$ , respectively:

$$\begin{aligned} \min_{d,x,g,y,s} \frac{\alpha}{2} \left\| s - \sum \sum d * x \right\|_2^2 + \lambda_1 \|y\|_1 + \frac{\lambda_2}{2} \|y\|_2^2 \\ + \frac{\gamma}{2} \|R\mathcal{F}_2s - m\|_2^2 + \frac{\rho}{2} \|x - y + u\|_2^2 \\ + \frac{\sigma}{2} \|d - g + h\|_2^2 \text{ s.t. : } g = Proj(d), \|g\|_2^2 \leq 1 \end{aligned} \quad (3)$$

We solve the problem (3) by iteratively finding the minimization solution of subproblems similar to the ADMM method as shown below:

**Solve for  $x$ :**

$$\min_x \frac{\alpha}{2} \left\| \sum \sum d * x - s \right\|_2^2 + \frac{\rho}{2} \|x - y + u\|_2^2 \quad (4)$$

The solution in the Fourier domain is shown in (5).

$$(\alpha D_f^H D_f + \rho I) x_f = D_f^H s_f + \rho(y_f - u_f) \quad (5)$$

where  $D_f$  is the concatenated of all diagonalized matrices  $d_{f_{n,k}}$  as illustrated in (6) and  $D_f^H$  is the Hermitian transpose of  $D_f$ .

$$D_f = [diag(d_{f_{1,1}}), \dots, diag(d_{f_{1,k}}), \dots, diag(d_{f_{n,k}})] \quad (6)$$

**Solve for  $y$ :**

$$\min_y \lambda_1 \|y\|_1 + \frac{\lambda_2}{2} \|y\|_2^2 + \frac{\rho}{2} \|x - y + u\|_2^2 \quad (7)$$

The single-scale 3D CSC [3] used only  $l_1$  regularization but in our subproblem, it contains both  $l_1$  and  $l_2$  regularizations. Fortunately, we can also solve it by using a shrinkage operation:

$$y = S_{\lambda_1/(\lambda_2+\rho)} \left( \frac{\rho(x+u)}{\lambda_2+\rho} \right) \quad (8)$$

**Update for  $u$ :**

The update rule for  $u$  is shown in (9)

$$u = u + x - y \quad (9)$$

**Solve for  $d$ :**

$$\min_d \frac{\alpha}{2} \left\| \sum \sum d * x - s \right\|_2^2 + \frac{\sigma}{2} \|d - g + h\|_2^2 \quad (10)$$

We solve this subproblem similar to  $x$ :

$$(\alpha X_f^H X_f + \sigma I) d_f = X_f^H s_f + \sigma(g_f - h_f) \quad (11)$$

Note that  $X_f$  stands for the concatenated matrix of all diagonal matrices  $x_{f \ n,k}$  as shown in (12) and  $X_f^H$  is the Hermitian transpose of  $X_f$

$$X_f = [\text{diag}(x_{f \ 1,1}), \dots, \text{diag}(x_{f \ 1,k}), \dots, \text{diag}(x_{f \ n,k})] \quad (12)$$

**Solve for  $g$ :**

$$\min_g \frac{\sigma}{2} \|d - g + h\|_2^2 \text{ s.t. : } g = \text{Proj}(d), \|g\|_2^2 \leq 1 \quad (13)$$

$g$  can be solved by using the inverse Fourier transform of  $d_f$ . This projection should be constrained by suppressing the elements that are outside the filter size  $d_{n,k}$ , and followed by normalizing its  $l_2$ -norm to a unit length.

**Update for  $h$ :** Similar to  $u$ , we update  $h$  as follows:

$$h = h + d - g \quad (14)$$

**Solve for  $s$ :**

$$\min_s \frac{\alpha}{2} \left\| s - \sum \sum d * x \right\|_2^2 + \frac{\gamma}{2} \|R \mathcal{F}_2 s - m\|_2^2 \quad (15)$$

Subproblem (15) can be transformed and solved in 2D Fourier domain as in [3]; therefore,  $s_{f_2}$  can be found by solving (16)

$$(\gamma R^H R + \alpha I) s_{f_2} = \gamma R^H m + \alpha \mathcal{F}_t^H \sum \sum d_f x_f \quad (16)$$

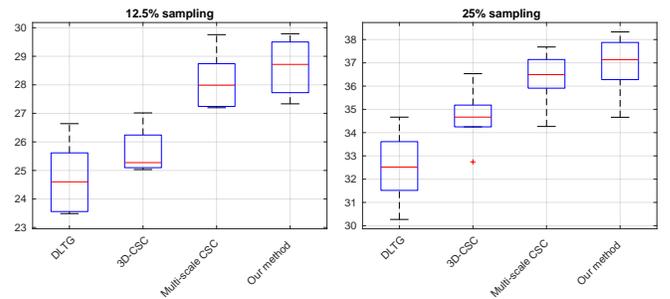
Note that we can efficiently solve independent linear systems (5), (11), and (16) via the Sherman-Morrison formula as shown in [5]. To the end, after the iteration process,  $s$  will be the results of applying a 2D inverse Fourier transform  $\mathcal{F}_t^H$  on  $s_{f_2}$ .

### 3. RESULTS

To evaluate the performance of our method, we used six cardiac MRI datasets from The Data Science Bowl [8]. Each dataset consists of 30 frames of a  $256 \times 256$  image across the cardiac cycle of a heart. In addition, as the above derivation consists of only the Fourier transform and element-wise operations, it can be accelerated on data-parallel architecture, such as GPUs. Thus, we used MATLAB 2017a to implement the prototype of the proposed method using the GPU. In the

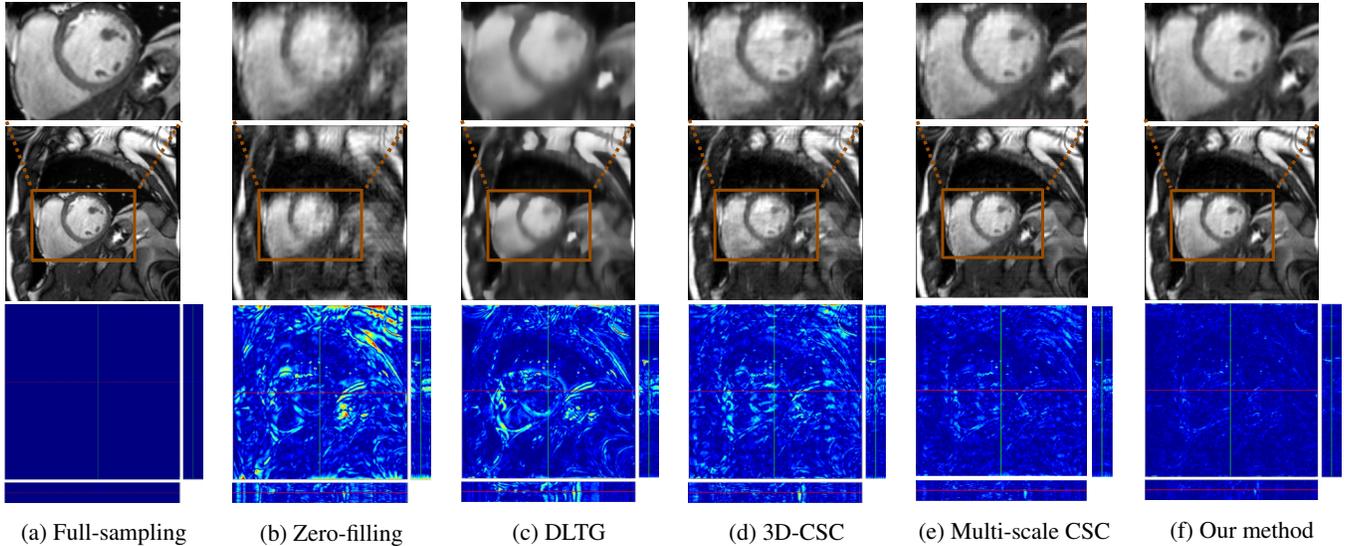
experiment, three different sizes of 3D filters were used with nine filters for each size (total 27 filters). The parameters used in our experiments are  $\alpha = 1.4$ ,  $\gamma = 0.07$ ,  $\lambda_1 = 0.03$ ,  $\lambda_2 = 4.9$ ,  $\rho = 95.4$ , and  $\sigma = 36.6$ , which are empirically found.

**Quality evaluation:** For a fair comparison, we used the same number of filters (27 filters) for all comparison methods: single-scale 3D CSC with recommended parameters in [3] (3D-CSC), multi-scale 3D CSC with only  $l_1$  regularization (multi-scale CSC), and Caballero et. al.'s method (DLTG) [4]. Our approach generates less errors than other methods do (see the region of interest, image quality, and color map for pixel-wise error reconstruction as shown in the first, second, and third rows, respectively, of Fig. 3). In addition, multi-scale 3D CSC can effectively reconstruct different features, especially in the heart region. The various sizes of learned atoms also capture the time trait better even under fast motion; therefore, they can reconstruct well the temporal features in the MR images. Moreover, the shift-invariance of CSC helps to generate more compact filters, compared with the patch-based method. The boxplots (Fig. 2) illustrate the archived PSNRs for 12.5% and 25% of the sampling masks. In our experiments, the PSNRs of the proposed method is significantly higher than 3D-CSC [3], and DLTG [4] and the elastic net regularization can improve the reconstruction quality, compared with using only  $l_1$  regularization in multi-scale 3D-CSC.



**Fig. 2:** PSNRs evaluation; unit: dB

**Running time evaluation:** We ran all the experiments on a PC equipped with an Intel i7-7700K CPU and a NVIDIA Titan X GPU, and measured the wall clock running times. In Table 1, our method running on the GPU achieved a speed-up of  $137 \times$ ,  $7.5 \times$ , and  $11 \times$  over DLTG [4], 3D-CSC (cpu) [3], and our method implemented on the CPU, respectively. However, the single-scale 3D-CSC is faster than our method because of the expensive computation of multiple dictionary sizes. We expect a further speed up using NVIDIA CUDA and C/C++ instead of MATLAB, which is left for the future work.



**Fig. 3:** 12.5% sampling reconstruction image quality and pixel-wise error comparison (blue: low, red: high)

	Running time (minute)
DLTG [4] (CPU)	232.1
3D-CSC [3] (CPU)	12.16
Our method (CPU)	18.21
3D-CSC [3] (GPU)	1.38
Our method (GPU)	1.62

**Table 1:** Average reconstruction time (200 epochs)

#### 4. CONCLUSION

In this paper, we have shown that the MRI reconstruction quality can be improved by using multi-scale 3D CSC because different filter sizes adapt well to features in various scales. Elastic net regularization also increased image quality compared to using  $l_1$  regularization only. Our method resulted in higher PSNRs than the other state-of-the-art single-scale 3D CSC and patch-based dictionary learning in cardiac dynamic MRI. In the future, we plan to develop a systematic approach to find an optimal set of parameters. Further accelerating the running times using high-performance computing systems is another future direction to explore.

#### 5. ACKNOWLEDGEMENT

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